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and if we add to this last equation after multiplying it by 3, equations (1) and (2), we obtain,

$$x^3+y^3+3xy(x+y)=16(x+y)-24=(x+y)^3.$$

If then $x+y=t$, $xy=u$, we have, $t^3-16t+24=0$, $u=8-24/t$. The cubic can be written,

$$(t-2)(t^2+2t-12)=0.$$

For $t=2$, we have $u=8-\frac{24}{2}=-4$, so that the corresponding values of x or y are roots of $z^2-2z-4=0$; hence one system of values is $x=1+\sqrt{5}$, $y=1-\sqrt{5}$, and another, by permuting x and y.

The other factor of the equation in t has for roots,

$$t_1=-1+\sqrt{13},\quad t_2=-1-\sqrt{13},$$

to which corresponds,

$$u_1=6-2\sqrt{13},\quad u_2=6+2\sqrt{13}.$$

To (t_1, u_1) correspond the roots of $z^2-(-1+\sqrt{13})z+6-2\sqrt{13}=0$, or

$$x=\sqrt{13}-1+\sqrt{[-6\sqrt{(13)}-10]},\quad y=\sqrt{13}-1-\sqrt{[-6\sqrt{(13)}-10]},$$

and the point obtained by permuting x and y.

To have the values of (x, y) corresponding to (t_2, u_2) it is sufficient to change the signs affecting $\sqrt{13}$, in the two preceding ones, and we obtain

$$x=\tfrac{1}{2}[-\sqrt{(13)}-1+\sqrt{[6\sqrt{(13)}-10]},$$
$$y=\tfrac{1}{2}[-\sqrt{(13)}-1-\sqrt{[-6\sqrt{(13)}-10]},$$

and the point obtained by permuting x and y.

Also solved by Jeannette Brooks, Nellie Wood, J. Scheffer, V. M. Spunar, A. H. Holmes, and the Proposer.

353. Proposed by DANIEL KRETH, Oxford, Iowa.

Divide 2940 into two such factors that the square of one factor minus 21 will equal three times the other factor.

Solution by J. K. ELLWOOD, Kansas City, Mo.

Let x be one factor, then $2940/x$ is the other.

By the conditions of the problem, $x^2-21=3\times2940/x=8820/x$.

Whence, $x^3-21x=8820$. Multiplying both sides of the equation by x, $x^4-21x^2=8820x$. Adding $441x^2$ to both sides, $x^4+420x^2=441x^2+8820x$.

Completing squares, $x^4+420x^2+210^2=441x^2+8820x+210^2$, or $(x^2+210)^2=(21x+210)^2$.

Whence, $x^2=21x$, $x=0$, the introduced root, 21, and $\frac{1}{2}[-21\pm\sqrt{(-1239)}]$. Hence, 21 is the only possible real value for x. Therefore, $2940/21=140$, the other factor. Hence 21 and 140 are the factors.

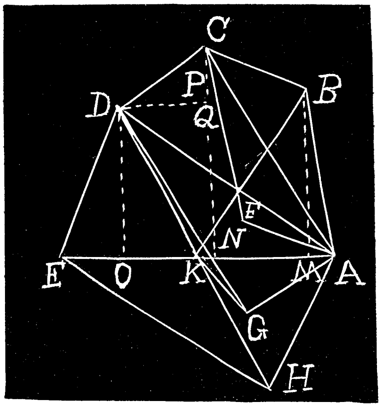
Also solved by H. Prime, V. M. Spunar, S. Lefschetz, J. Scheffer, and S. G. Barton.

GEOMETRY.

377. **Proposed by S. A. COREY, Hiteman, Iowa.**

Let AB, BC, CD, DE, EA be the sides of a pentagon, plane or gauche. From A draw AF, AG, AH, parallel to, and of the same length and currency as BC, CD, DE, respectively. Bisect AE at K. Draw KB, KF, KG, and KH. Prove that $KB^2+KF^2+KG^2+KH^2=AB^2+BC^2+CD^2+DE^2$.

I. **Solution by J. SCHEFFER, A. M., Hagerstown, Md.**



K being the mid-point of the diagonal AE in the parallelogram $ADEH$, DKA must be a straight line, viz., the other diagonal. Let the orthogonal coordinates of B, F, G, H, with reference to AE as the axis of X, and A as origin, be, respectively, x_1, y_1; x_2, y_2; x_3, y_3; x_4, y_4; and $AK=EK=a$. Then

$$AB^2+AF^2+AG^2+AH^2=(x_1^2+y_1^2)+(x_2^2+y_2^2)+(x_3^2+y_3^2)+(x_4^2+y_4^2)\text{.....(I)},$$

and $KB^2+KF^2+KG^2+KH^2$

$$=[(x_1-a)^2+y_1^2]+[(x_2-a)^2+y_2^2]+[(x_3-a)^2+y_3^2]+[(x_4-a)^2+y_4^2]$$
$$=(x_1^2+y_1^2)+(x_2^2+y_2^2)+(x_3^2+y_3^2)+(x_4^2+y_4^2)$$
$$-2a[(x_1+x_2+x_3+x_4)-2a]\text{...(II)}.$$

Letting fall the perpendiculars BM, CN, DO upon AE, and BP, DQ upon CN, we have $AB\cos BAM+BC\cos CBP+CD\cos CDA+DE\cos DEO=AB\cos BAM+AF\cos FAK+AG\cos KAG+AH\cos KAH=AE=2a$.

$\therefore x_1+x_2+x_3+x_4=2a$; therefore in (II), $x_1+x_2+x_3+x_4-2a=0$, and $AB^2+AF^2+AG^2+AH^2=KB^2+KF^2+KG^2+KH^2=AB^2+BC^2+CD^2+DE^2$. Q. E. D.